THERMOMOLECULAR PRESSURE DIFFERENCE AT LOW KNUDSEN NUMBERS

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The experimental dependence of the thermomolecular pressure difference on the average pressure is found for He, Ne, H_2, and D_2 in the viscous-flow regime ($Kn^{-1} = 100-500$). The experimental data are compared with analytic expressions in the literature.

The thermomolecular pressure difference which arises in a gas under nonisothermal conditions is of both practical and intrinsic interest, since it is one of the classical problems of the dynamics of a low-density gas. It can be used in tests of various methods for solving the basic kinetic equation and its models as well as in tests of various methods of the thermodynamics of irreversible processes. Fundamental differences among the theoretical approaches to this problem have led to a wide variety of equations describing this effect in the limiting viscous-flow regime. The results calculated from the various equations differ by more than two orders of magnitude.

The lack of experimental data for this type of flow has prevented a resolution of these discrepancies. The data are lacking primarily because of the difficulties which arise in attempts to accurately measure very small pressure drops, $\sim 10^{-1}$ N/m^2.

We report here a measurement of the thermomolecular pressure differency by the comparison method of [1], as modified in a study [2] of this effect in an intermediate flow regime. The thermomolecular pressure difference is measured directly by a differential microgauge with digital readout [3]. The instrument has a sensitivity $\sim 3 \cdot 10^{-4}$ (N/m^2)/Hz.

The detection method is shown schematically in Fig. 1. The upper part of the apparatus, which includes the microgauge 1, the evacuation system, the inlet system, and the system for measuring absolute pressures (I), is at room temperature. The lower part of the working chamber (the "cold-gas" chamber) is in a Dewar filled with liquid nitrogen. Because of the large ratio of diameters (1 : 200) of capillary 3 and compensation tube 6, the thermomolecular pressure difference in the wide tube is negligible.

Instead of a single capillary we actually use a packet of 100 glass capillaries ~ 0.015 cm in radius and ~ 25 cm long. The scatter in the capillary diameters around the average value does not exceed 1%. The capillary diameter and length are measured by an optical method.

TABLE 1. Values of the Constant A

Method	A
Thermodynamics of irreversible processes [4]	6,1
The same [5]	4,0
Elementary theory [6]	3,36
BGK model [7]	3,06
Experimental [8]	3,86; 4,15
Experimental (the present study)	3,02±0,04

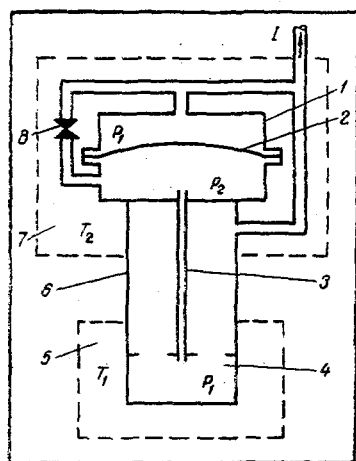<div style="text-align:center">Fig. 1   Fig. 2</div>

Fig. 1. System used to measure the thermomolecular pressure difference. 1) Microgauge; 2) membrane; 3) capillary; 4) "cold-gas" chamber; 5) cryostat; 6) compensation tube; 7) thermoregulator; 8) bypass valve.
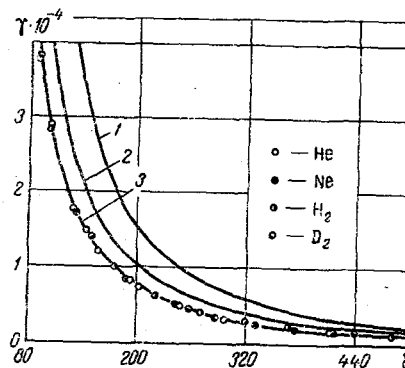
Fig. 2. Experimental and theoretical results. 1) [4]; 2) [5]; 3) [7].

After bypass value 8 is closed a thermomolecular pressure difference is gradually established in channel 3 in a transient process. This difference is determined from the difference between the indications of the frequency meters with the bypass valve in the open and closed positions, through the use of the microgauge calibration.

We measured the pressure drop ΔP_{TMD} as a function of the pressure in the cold chamber for several gases: He, Ne, H_2, and D_2. Pressure P_1 is measured by a mercury gauge and is varied from a few tenths to hundreds of torr. The data are converted to dimensionless form to facilitate a comparison of the data for the different gases and a comparison with theory.

Analysis of the available theoretical results shows that the ratio of the pressures in the hot and cold chambers, connected by the capillary, can be written as

$$\frac{P_1}{P_2} = \left(\frac{T_1}{T_2} \right)^{\gamma},$$ (1)

where γ is a function of the Knudsen number alone and is governed by the average temperature and the pressure.

The familiar Knudsen equation for the free-molecular limit can be obtained from (1) by setting $\gamma = 1/2$. It is not difficult to show that in the viscous-flow regime the value of γ tends toward zero according to

$$\gamma = A \frac{1}{\delta^2}, \quad \delta = \frac{\sqrt{\pi}}{2} \cdot \frac{R}{\lambda}.$$ (2)

A method of the thermodynamics of irreversible processes was used in [4, 5] to obtain equations which can be applied to the problem under consideration. An analogous equation (agreeing with this one within a constant factor) was derived from elementary kinetic theory in [6]. Sone and Yamamoto [7] derived an equation for the thermomolecular pressure difference in the viscous-flow regime by solving the Boltzmann equation with the Bhatnager—Gross—Krook model for the collision operator. All these theoretical results were reduced to form (2) through the use of the gas of hard-sphere molecules in the Chapman—Enskog approximation.

Table 1 shows the values of the coefficient A in Eq. (2) obtained from various theoretical [4-7] and experimental [8] studies; also shown here is the value of A found in the present study and averaged over the gases used.

Figure 2 compares the theoretical and experimental results in a plot of γ as a function of δ.

In analyzing the data we assumed δ to be governed by the arithmetic average of the pressures in the hot and cold chambers and by the arithmetic average of the corresponding temperatures.

This comparison shows that the experimental data are described best by the equation derived by Sone and Yamamoto [7]. In this case the discrepancy between theory and experiment is ~1-2% over the range of Knudsen numbers studied.

There is undoubtedly some arbitrariness involved in the choice of the experimental parameters (the average pressure and the temperature) governing δ, but calculations show that a change in the method for averaging these parameters—in particular, an account of the temperature dependence of the viscosity of real gases and the display of the experimental and theoretical results in integral form [3] — does not significantly change the results of the comparison of theory and experiment (it simply becomes more difficult to compare the data for the various gases).

NOTATION

Kn	is the Knudsen number;
P_1, P_2	are the pressures in the "cold" and "hot" chambers of the system;
T_1, T_2	are the corresponding absolute temperatures;
γ	is a function of the Knudsen number;
δ	is a dimensionless parameter;
A	is the numerical constant;
R	is the capillary radius;
λ	is the molecular mean free path;
ΔP_{TMD}	is the thermomolecular pressure difference.

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